On a : $V = \pi\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} + \pi.\lambda + 2\pi J$

$V = \left(\frac{\pi^4}{24} + \frac{\pi^2}{4} + 2\pi\right)$ unité de volume

Solution 12 :

1°/ $D_f = \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi \text{ où } k \in \mathbb{Z}\right\}$

2°/ $A = \int_0^{\frac{\pi}{4}} |f(x) - 1| \, dx = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} dx = \int_0^{\frac{\pi}{4}} (1 + tg^2 x - 1) dx$

$= [tgx - x]_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$

$A = \left(\frac{4-\pi}{4}\right) uA$

3°/ a)

- $\varphi'(x) = \frac{\cos^4 x + \sin^2 x (3\cos^2 x)}{\cos^6 x} = \frac{1}{\cos^2 x} + 3\frac{\sin^2 x}{\cos^4 x}$

$\varphi'(x) = \frac{1}{\cos^2 x} + 3\frac{(1 - \cos^2 x)}{\cos^4 x}$

$\varphi'(x) = \frac{3}{\cos^4 x} - \frac{2}{\cos^2 x} = 3f^2(x) - 2f(x)$

b) $V = \pi \int_0^1 f^2(x) dx = \pi \int_0^{\frac{\pi}{4}} \frac{1}{\cos^4 x} dx$

On a : $f^2(x) = \frac{\varphi'(x) + 2f(x)}{3}$

D'où : $V = \frac{\pi}{3} \int_0^{\frac{\pi}{4}} (\varphi'(x) + 2f(x)) \, dx$

$V = \frac{\pi}{3} [\varphi(x)]_0^{\frac{\pi}{4}} + \frac{2\pi}{3} \int_0^{\frac{\pi}{4}} f(x) \, dx$